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Robustness of fuzzy control and its application to a thermal plant

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Abstract

The paper deals with the robustness of fuzzy control. The complementary sensitivity function which in the case of the cancellation control corresponds to the desired closed-loop transfer function must be shaped in order to fulfil the basic equation of robust stability. With classical robust control this leads to poor performance. Fuzzy models are treated as a superset of linear models which are softly switched. It is supposed that every linear model contributes to the output according to the fulfilment grade vectors of membership functions on the universe of discourse. Under this supposition the multiplicative uncertainty of the model is reduced and a better control can be achieved. The proposed procedure is first illustrated on a simulated first order nonlinear process and then applied to a thermal plant — the laboratory scale heat exchanger.

Keywords: Intelligent control; Fuzzy control; Cancellation control; Nonlinear control; Heat exchangers

1. Introduction

The paper deals with the robustness of the fuzzy model based cancellation control. It has been shown that under certain conditions fuzzy controllers are a superset of linear controllers. The same is valid for fuzzy models: they are a superset of linear models. Actually, nonlinear models — described by fuzzy models — can be interpreted as consisting of a set of linear models which are softly switched. Every linear model contributes to the output according to the fulfilment grade vectors of membership functions on the universe of discourse. Fuzzy models as used in this paper were proposed by Takagi and Sugeno [10] where consequent proposition is a crisp function of the antecedent variables. An overview of fuzzy modelling for control is reviewed in [2]; different approaches to fuzzy controllers are given in [4]. It has been shown that fuzzy logic and neural nets are closely related; actually, it has been shown that a class of fuzzy logic systems — the learning of fuzzy rules from numerical data — is equal to the training of radial basis function [6]. Classical cancellation controllers were introduced in [8] and widely used under different names (see Isermann et al. [3]) in [1,7]. Fuzzy adaptive cancellation control was introduced in [9]. Stability, robustness and performance of fuzzy controllers is discussed in [5].

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In this paper the robust stability is used for designing the desired performance of the closed loop behaviour using the cancellation controller. The paper is organized as follows: first the fuzzy models are discussed in Section 2. Model based cancellation control is presented in Section 3 while robustness of the proposed control scheme is discussed in Section 4. The superior performance of the fuzzy cancellation control versus classical cancellation control is shown in Section 5 by simulation. An application of the proposed control algorithm on a thermal plant is presented in Section 6.

2. Fuzzy models

A typical Takagi–Sugeno type rule can be written:

$$R^j: \quad \text{if} \quad x_1 \text{ is } A_1^j \text{ and} \ldots \text{and } x_n \text{ is } A_n^j \quad \text{then} \quad y = f^j(x_1 \ldots x_n) \tag{1}$$

where $x_1 \ldots x_n$ are inputs, A_1^j a subset of the input space, y output and f^j a function (in general nonlinear, usually linear). If f^j is constant, the resulting Takagi–Sugeno model is called Takagi–Sugeno model with crisp constant consequences [2,10]:

$$R^j: \quad \text{if} \quad x_1 \text{ is } A_1^j \text{ and} \ldots \text{and } x_n \text{ is } A_n^j \quad \text{then} \quad y = r_{i1,i2,\ldots,in} \tag{2}$$

The output of the fuzzy model with n inputs can be written in the following form:

$$y(k) = \frac{\sum_{i1}, \sum_{i2}, \ldots, \sum_{in} s_{i1,i2,\ldots,in}(\boldsymbol{x}) r_{i1,i2,\ldots,in}}{\sum_{i1}, \sum_{i2}, \ldots, \sum_{in} s_{i1,i2,\ldots,in}} \tag{3}$$

where $s_{i1,i2,\ldots,in}$ is the element of the multidimensional structure:

$$S = \mu_1 \otimes \mu_2 \otimes \ldots \otimes \mu_n \tag{4}$$

which is obtained by the composition $\otimes$ (usually a product or an other T norm) of fulfilment grade vectors (of dimensions mi) of membership functions on the universe of discourse:

$$\mu_i = [\mu_i^1, \mu_i^2 \ldots \mu_i^{mi}]^{\mathrm{T}}. \tag{5}$$

where mi is the number of membership functions. In Eq. (3), $\boldsymbol{x}$ stands for the regressor:

$$\boldsymbol{x}^{\mathrm{T}} = [u(k), \ldots, u(k - N_p), y(k-1), \ldots, y(k - N_p)] \tag{6}$$

$r_{i1,i2,\ldots in}$ are consequences of the Takagi–Sugeno type model according to Eq. (2). In the simplest case they are constants (Takagi–Sugeno models with crisp constant consequences).

If the antecedent membership functions are triangular and form a partition, i.e. $\sum_{j=1}^{mi} \mu_i^j = 1, \forall i$ the denominator of Eq. (3) becomes 1.

3. Model based cancellation control

Models are inherently involved in the design of controllers. With simple controllers, such as proportional-integral-derivative (PID) controllers and their fuzzy pendant in the form of a few simple rules, the models are used in tuning to replace costly experiments. However, in a strict sense the term ‘model based control’ denotes the group of control algorithms that involve the model of the controlled process (or a part of it) explicitly.

The classical model based controllers are cancellation controllers [8] which include the inverse of the process. The cancellation of process dynamics (its poles and zeros in the linear case) is an essential part of perfect linear model following [7] and of pole zero placement [1] controllers.

The idea of classical cancellation controllers is to compensate the dynamic behaviour of the plant, and thus force the plant output to follow the output of a prescribed model incorporating the desired input–output relation of the closed loop system [8]. The same idea can be applied to nonlinear plants which are supposed to be described by fuzzy models [5,9].

After the plant dynamic has been compensated to a pure time delay, the second part of the controller is designed in the same way as in the linear case. Also, linear models are usually chosen to be the reference models (models to be followed). In order to avoid prediction, they must have at least the same delay as the controlled plant. Since additional delays would increase the delay in the closed loop system, the time delay of the prescribed (reference) model is chosen to be the same as the time delay of the controlled plant.

The essential part of the fuzzy cancellation controller is a compensator which compensates the nonlinear dynamic behaviours of the plant.

4. Robustness issues of fuzzy model based cancellation control

It has been shown that under certain conditions [4] fuzzy controllers are a superset of linear controllers. The same is valid for fuzzy models: they are a superset of linear models. Actually, nonlinear models — as treated in this paper and fulfilling the criteria 1–8 of [4] — can be interpreted as consisting of a set of linear models and a soft switch. Every linear model contributes to the output according to the multidimensional structure $\boldsymbol{S}$, i.e. according to the fulfilment grade vectors of membership functions on the universe of discourse.

In classical robust control, a nominal plant model is chosen and a constant controller is designed to satisfy the basic equation of the robust stability:

$$\|\Delta W T\|_{\infty} < 1 \tag{7}$$

where T is the complementary sensitivity function (the transfer function of the closed loop), ΔW the multiplicative uncertainty of the plant and $\| \; \|_{\infty}$ the ∞ norm (the least upper bound of the absolute value).

If the actual plant is presumed to consist of a set of linear models (which might be diversified with respect to the gain and the time constants) the multiplicative uncertainty is large. The nominal model is chosen in such a way that the multiplicative uncertainty is less than one at low frequencies, while at higher frequencies the fulfilment of Eq. (7) is assured by shaping the complementary sensitivity function T. Since the multiplicative uncertainty of the plant is large, the complementary sensitivity function must be small. This results in poor performance by the classical robust control.

In the case of fuzzy models the controller consists of a set of controllers which correspond to the set of models. Every controller contributes to the control variable with regard to the contribution of the corresponding model. Since the uncertainty of the individual linear models is much smaller than the uncertainty of the entire nonlinear model, the complementary sensitivity function may be larger, and better control performance can be achieved. It should be noted, however, that due to the nonlinearities,

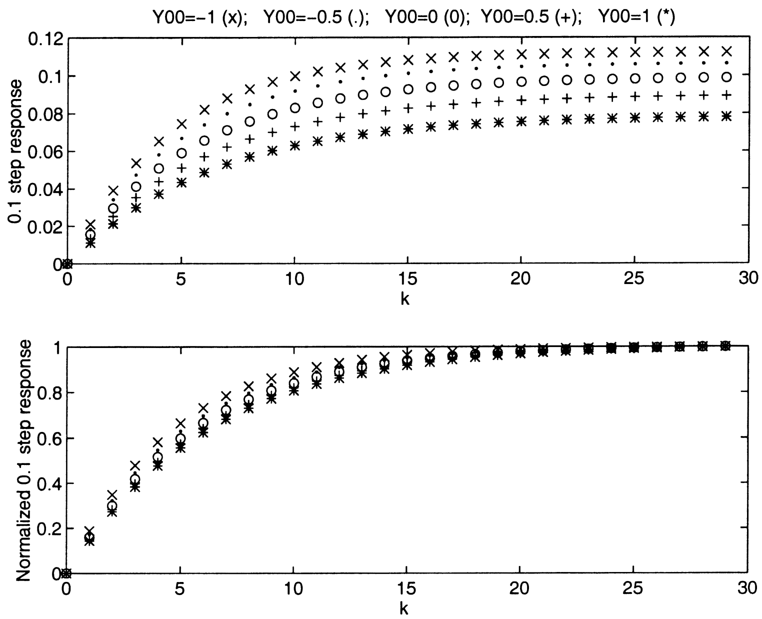

Fig. 1. Step responses in various operating points.

the proposed robustness analysis is valid only locally. Nevertheless, as shown later it can successfully contribute to the design procedure.

5. Fuzzy robust control of a nonlinear process

The effectiveness of the fuzzy control in comparison with classical robust control will be illustrated on an example of the discrete time nonlinear process described by the following differential equation:

$$y(k) = 0.8930y(k-1) + 0.0371y^2(k-1) - 0.05y(k-2) + 0.157u(k) - 0.05u(k)y(k-1) \quad (8)$$

where $u(k)$ and $y(k)$ are the control and controlled variables, respectively. The controlled variable is supposed to remain in the interval $(-1, 1)$. Fig. 1 depicts the open loop step responses of the process in different operating points (steady state values of the controlled variable $Y_{00} = -1, -0.5, 0, 0.5$ and 1).

In the upper part of the figure the responses to the step with the amplitude 0.1 are depicted while the lower part of the figure shows the same responses normalised with respect to the current static gain. It can be seen that the nonlinear process exhibits approximately first order dynamics with noticable variations of the gain and the time constant. For the classical robust controller a first order nominal process with the following transfer function was chosen:

$$G_o(z) = \frac{0.107}{1 - 0.893z^{-1}} \quad (9)$$

The process uncertainty was calculated as a maximum of the absolute value of the frequency response deviations between the nominal process and linearised processes in the five operating points. In Fig. 2

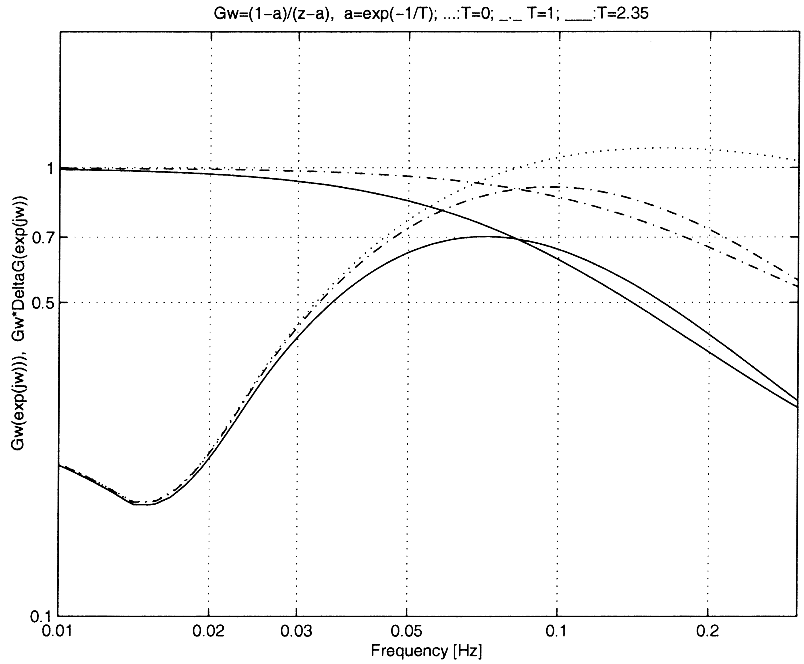

Fig. 2. The robust stability criterion for the classical robust control.

the multiplicative uncertainty is depicted by the dotted line. According to the classical robust stability criterion the product of the multiplicative uncertainty and the complementary sensitivity function must be less than 1. Since the multiplicative uncertainty in greater than 1 in the frequency range 0.08–0.4 Hz, the complementary sensitivity function must be shaped in order to satisfy the robust stability criterion. Fig. 2 depicts the product of the multiplicative uncertainty and the closed loop transfer function (complementary sensitivity function). Dash-dotted and solid lines represent the product of the uncertainty with the complementary sensitivity function as a first order lag with the time constants 1 and 2.05 s, respectively. It can be seen that the shaping of the closed loop transfer function robustly stabilises the closed loop system in the presence of process uncertainty. The first order lag with the time constant of 2.05 s was chosen as the prescribed closed loop behaviour for the design of classical cancellation controller since it satisfies the stabilisation criterion with a 3 dB margin.

Next the nonlinear process was identified as a Takagi–Sugeno fuzzy model with first order consequences. The resulting model is:

$$R^1: \quad \text{if} \quad y(k) \text{ is } A^1 \quad \text{then} \quad y(k+1) = 0.7965y(k) + 0.2171u(k) \tag{10}$$

$$R^2: \quad \text{if} \quad y(k) \text{ is } A^2 \quad \text{then} \quad y(k+1) = 0.8766y(k) + 0.1101u(k) \tag{11}$$

where A^1 and A^2 are subsets with membership functions depicted in Fig. 3 The frequency response of the fuzzy model was calculated as a weighted sum of frequency responses of models (10) and (11) according to their fulfilment grade. The uncertainty of the fuzzy model was evaluated as the difference between the frequency responses of the linearised process and the frequency response of the fuzzy model.

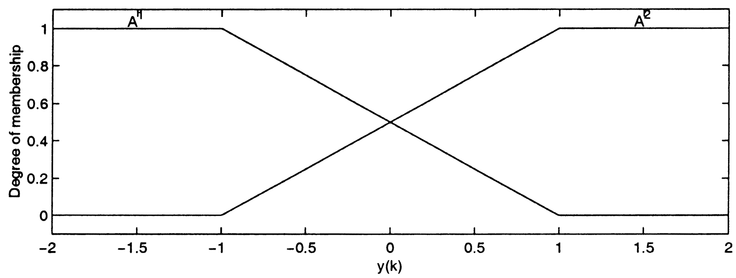

Fig. 3. Membership functions.

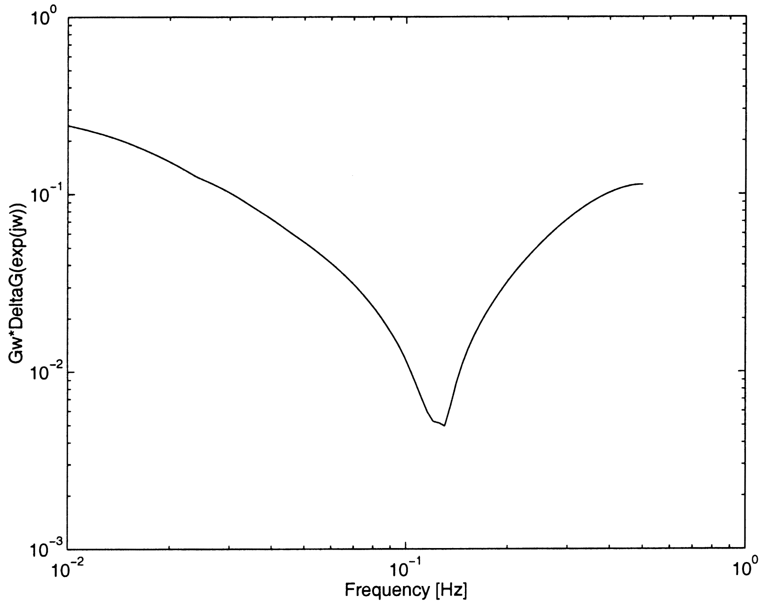

Fig. 4. The multiplicative uncertainty for the NF model.

The multiplicative uncertainty of the fuzzy model is shown in Fig. 4 where it can be seen that it is less than 1 (actually less than 0.25) in the entire frequency range. Therefore no shaping of the complementary sensitivity function is required and a one step delay was chosen as the prescribed closed loop behaviour. The fuzzy compensation controller was realised as the combination of two compensation controllers for linear processes (10) and (11). The fuzzy control signal is the weighted sum of both control signals according to their fulfilment grade of the premise.

Fig. 5 depicts the closed loop response to the 0.1 step in three different operating points: $Y_{00} = -1$ (denoted by x) $Y_{00} = 0$ (denoted by o) and $Y_{00} = +1$ (denoted by +) for the classical robust controller (upper part) and for the robust fuzzy controller (lower part). It can be seen that the fuzzy response is faster and is less dependent on the operating point. It can be concluded that the fuzzy modelling technique (as a soft switching among different linear models) can substantially contribute to the improvement of the robustness and performance of the control of nonlinear systems.

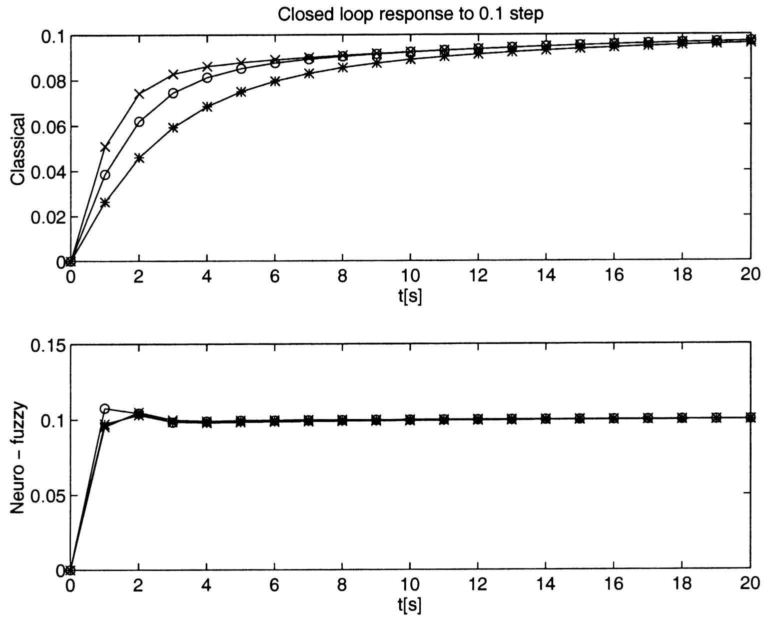

Fig. 5. The closed loop responses for the classical robust control (upper part) and for the NF control (lower part).

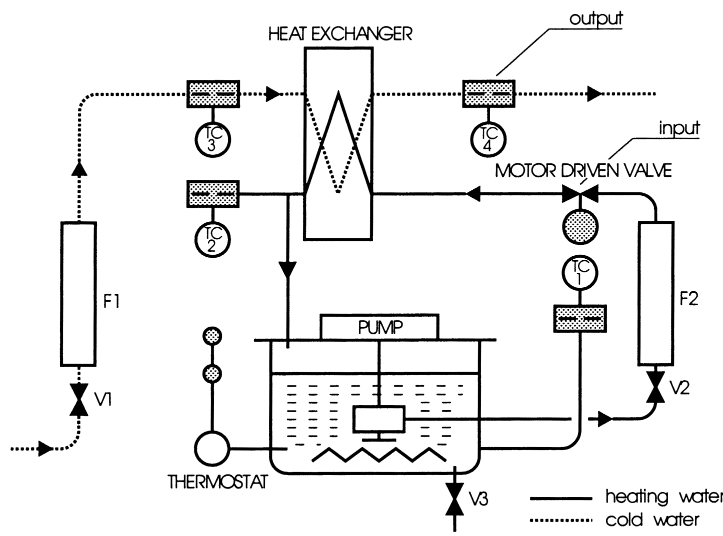

Fig. 6. The scheme of the laboratory scale heat exchanger.

6. Fuzzy model based cancellation control of a thermal plant

The proposed robust model based cancellation control has been tested on a thermal plant — a laboratory scale heat exchanger. The accessory, depicted schematically in Fig. 6, consists of a plate heat exchanger, through which hot water from an electrically heated reservoir is continuously circulated in counter-current flow to the cold process fluid (cold water). Thermocouples are located in the inlet and

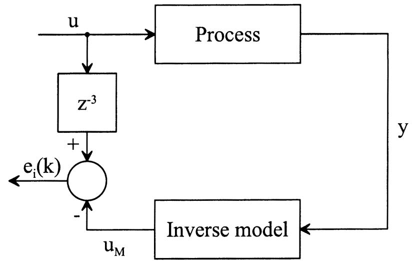

Fig. 7. The identification scheme.

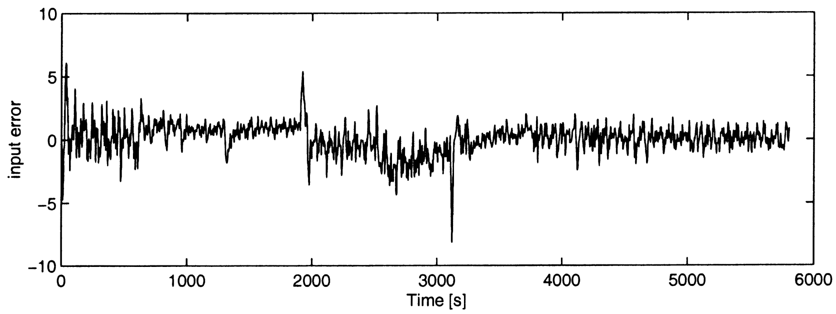

Fig. 8. The input error.

outlet streams of the exchanger, while the primary and secondary flow rates can be visually monitored. Power to the heater is controlled by an external control loop. The flow of the heating fluid can be controlled by a proportional motor driven valve. The control variable is the control current of the valve (4–20 mA), while the controlled variable is the temperature of the water in the secondary circuit at the heat exchanger outlet.

The plant was identified on the basis of signals which assure a throughout excitation. The sampling time used was 4 s. As the dynamics of the plant exhibits approximately first order dynamics with a small time delay, the input error was evaluated by:

$$e_i(k) = u(k-3) - u_M(k) \tag{12}$$

where the time delay of the plant was taken into account and the inverse of the plant without the time delay was identified by:

$$u_M(k) = f(y(k), y(k-1)) \tag{13}$$

Figs. 7 and 8 depict the identification scheme and the input error for the heat exchanger, respectively.

For the fuzzy model (1) the universe of discourse of the $y(k)$ and $y(k-1)$ variables were divided into five triangular equally spaced membership functions using the Takagi–Sugeno model with crisp constant consequences ($f^j = c^j$). The central values of the membership functions were [13.00, 22.75, 32.50, 42.25, 52.00]°C, respectively. The product was used as the composition operator $\otimes$, and the rule base was complete.

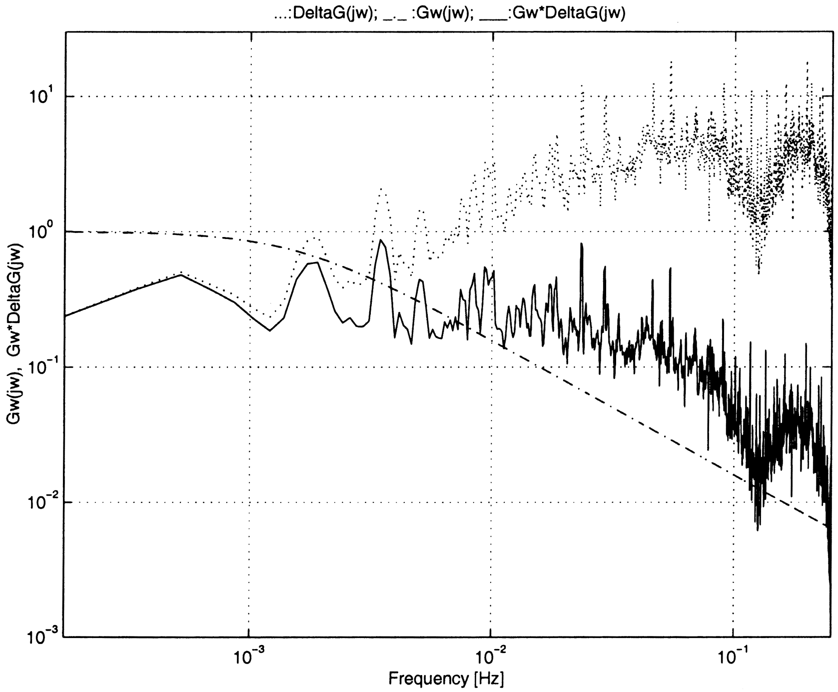

Fig. 9. The multiplicative uncertainty (- - -) the complementary sensitivity function (- · · -) and the product of both (—) for the fuzzy model based cancellation controller.

The next step is the design of a robust controller. For the purpose of design the multiplicative uncertainty is evaluated by the following procedure. If the process with its multiplicative uncertainty is written as a linear model in the form:

$$G_\mathrm{p} = G_\mathrm{o} z^{-d}(1 + \Delta G_\mathrm{p}) \tag{14}$$

and the identified model is the inverse of the nominal process without the time delay (G_o^{-1}) the absolute value of the transfer function from the control variable u to the input error e_i equals to the multiplicative uncertainty of the process (ΔG_p). It can be estimated by dividing the Fourier transforms of the input error and the control variable. The same procedure can be used for fuzzy models if the nonlinear function realised by fuzzy rules is supposed to be injective.

Using this procedure the multiplicative uncertainty of the plant was estimated and is illustrated in Fig 9 by the dotted line. Since it is greater than 1 in the frequency range above 0.003 Hz the desired closed loop (and consequently the complementary sensitivity function) must be shaped in order to fulfil the robust stability criterion (7). In Fig 9 the complementary sensitivity function T is depicted by dash-dotted line and the product $T \times \Delta G_\mathrm{p}$ by solid line, respectively. The required cut-off frequency of the complementary sensitivity function was 0.0016 Hz (which corresponds to a first order lag with the time constant 100 s).

Fig. 10 depicts the closed loop response of the fuzzy model-based cancellation control. It can be seen that the closed loop response remains approximately the same throughout the entire range of the controlled variable, and that the response is stable and adequate.

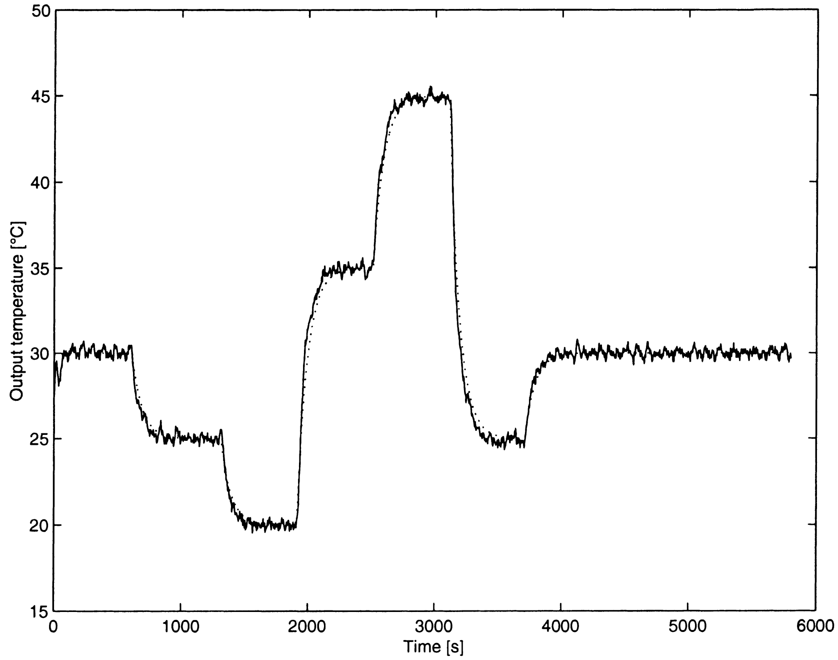

Fig. 10. The closed loop responses (the desired model (- - -); the plant (—)) of the fuzzy model based cancellation control of the heat exchanger.

7. Conclusion

The robustness of fuzzy cancellation controllers was discussed and first illustrated on the example of a nonlinear model. It was shown that robust fuzzy control exhibits superior performance with respect to the classical robust control. A procedure for the estimation of the multiplicative uncertainty of the input error model was proposed and applied to the robust fuzzy model based cancellation control of a laboratory scale heat exchanger.

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